ORM2: formalisation and encoding in OWL2

Enrico Franconi, Alessandro Mosca, and Dmitry Solomakhin
KRDB Research Centre
Free University of Bozen-Bolzano, Italy

Summary

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- A full formalisation of ORM2

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- A well founded and provably correct encoding in OWL2 of a relevant fragment of ORM2
- A Visual Studio plugin extending NORMA with reasoning

A formalisation of ORM2

- We need a precise and complete syntax and semantics
- We have two proposals (proved to be consistent wrt the original Halpin's formalisation, and equivalent among each other)
- We could build upon our formalisation
- Maybe we got something wrong, let's fix it

A correct encoding in OWL2

- A provably correct encoding in OWL2 of a relevant fragment of ORM2
- All the other approaches in the literature proved to be wrong
- We can extend the expressivity of the currently captured fragment

A Visual Studio plugin

- Still at an early stage of development, but already quite cool
- Loose coupling with NORMA
- We aim at a much tighter integration with NORMA;
  we would like to discuss how with you

Demo!

A formalisation of ORM2

Signature

- A set $\mathcal{E}$ of *entity type* symbols;
- a set $\mathcal{V}$ of *value type* symbols;
- a set $\mathcal{R}$ of *relation* symbols;
- a set $\mathcal{A}$ of *role* symbols;
- a set $\mathcal{D}$ of *domain* symbols, and
- a set Λ of pairwise disjoint sets of values;
- for each $D \in \mathcal{D}$, an injective extension function $\Lambda_{(\cdot)} : \mathcal{D} \rightarrow \Lambda$ associating each domain symbol D to an extension Λ_D;
- a binary relation $\varrho \subseteq \mathcal{R} \times \mathcal{A}$ linking role symbols to relation symbols. We take the pair $R.a$ as the atomic elements of the syntax, and we call it *localised role*. Given a relation symbol R, $\varrho_R = \{R.a | R.a \in \varrho\}$ is the set of localised roles with respect to R; $arity(R) = |\varrho_R|$ is the arity of the relation R;
- for each relation symbol R, a bijection $\tau_R \colon \varrho_R \rightarrow [1..|\varrho_R|]$ mapping each element in ϱ_R to an element in the finite sequence of natural numbers $[1..|\varrho_R|]$. We also define $\tau = \bigcup_{R \in \mathcal{R}} \tau_R$. The mapping τ_R guarantees a correspondence between role components and argument positions in a relation, so that we can freely choose between an 'attribute-based' and a 'positional-based' representation.

First-Order Logic

(i) $E_1, E_2, \ldots, E_n$ 1-ary predicates for *entity types*;
(ii) $V_1, V_2, \ldots, V_m$ 1-ary predicates for *value types*;
(iii) $D_1, D_2, \ldots, D_l$ 1-ary predicates for *domain symbols*;
(iv) $R_1, R_2, \ldots, R_k$ n-ary predicates for *relations*;
(v) a countable set of *constants* $d_1, d_2, \ldots$;
(vi) a set $\mathrm{ID}^2, \ldots, \mathrm{ID}^{n_{max}}$ of functions, $n_{max} = \mathsf{max}\{|\varrho_R| | R \in \mathcal{R}\}$.

– Background domain axioms:

$$\forall x.E_i(x) \rightarrow \neg(D_1(x) \vee \cdots \vee D_l(x)),\ \text{for } 1 \leq i \leq n \tag{1}$$

$$\forall x.V_i(x) \rightarrow D_j(x),\ \text{for } 1 \leq i \leq m \tag{2}$$

$$\forall x.D_i(x) \leftrightarrow (x = d_1 \vee x = d_2 \vee \ldots),\ \text{for all } d_i \in \Lambda_{D_i} \tag{3}$$

$$\forall x_1, \ldots, x_n, z_1, \ldots, z_n.\mathrm{ID}(\overline{\mathbf{x}}) = \mathrm{ID}(\overline{\mathbf{z}}) \leftrightarrow \overline{\mathbf{x}} = \overline{\mathbf{s}},\ \text{for } n = 1, \ldots, n_{max} \tag{4}$$

First-Order Logic

■ $\mathsf{TYPE} \subseteq \varrho \times (\mathcal{E} \cup \mathcal{V})$

□ If $\mathsf{TYPE}(R.a, O) \in \Sigma$ then $\forall x_1 \ldots x_{\tau(R.a)} \ldots x_n.R(x_1, \ldots, x_{\tau(R.a)}, \ldots, x_n) \rightarrow O(x_{\tau(R.a)})$

■ $\mathsf{FREQ} \subseteq \wp(\varrho) \times (\wp(\varrho) \times \wp(\varrho)) \times (\mathbb{N} \times (\mathbb{N} \cup \{\infty\}))$

□ If $\mathsf{FREQ}(\{R^1.a_{11}, \ldots, R^1.a_{1n}, \ldots, R^k.a_{k1}, \ldots, R^k.a_{km}\}, \bowtie_{\mathbf{R}}, (\mathsf{min}, \mathsf{max})) \in \Sigma$ then

$$\forall \overline{y}[\exists \overline{x}^1 \ldots \overline{x}^k (\bigwedge_{j=1}^{k} R^j(\overline{x}^j) \wedge \bigwedge_{\mathbf{i1}=1}^{n} (x^1_{\tau(R^1.a_{1\mathbf{i1}})} = y_{1\mathbf{i1}}) \wedge \cdots \wedge \bigwedge_{\mathbf{ik}=1}^{m} (x^1_{\tau(R^1.a_{1\mathbf{ik}})} = y_{1\mathbf{ik}}) \wedge \bigwedge_{\bowtie_{\mathbf{R}}} (x^{r^+}_{\tau(R^{r^+}.a_{r^+\mathbf{v}_r})} = x^{r^-}_{\tau(R^{r^-}.a_{r^-\mathbf{w}_r})}))] \rightarrow$$

$$\exists^{\geq \mathsf{min}; \leq \mathsf{max}} \overline{y}[\exists \overline{x}^1 \ldots \overline{x}^k (\bigwedge_{j=1}^{k} R^j(\overline{x}^j) \wedge \bigwedge_{\mathbf{i1}=1}^{n} (x^1_{\tau(R^1.a_{1\mathbf{i1}})} = y_{1\mathbf{i1}}) \wedge \cdots \wedge \bigwedge_{\mathbf{ik}=1}^{m} (x^1_{\tau(R^1.a_{1\mathbf{ik}})} = y_{1\mathbf{ik}}) \wedge (\bigwedge_{\bowtie_{\mathbf{R}}} x^{r^+}_{\tau(R^{r^+}.a_{r^+\mathbf{v}_r})} = x^{r^-}_{\tau(R^{r^-}.a_{r^-\mathbf{w}_r})}))]$$

where:

(1) $\bowtie_{\mathbf{R}} = \{\ldots, \langle R^i.a_{i\mathbf{v}} = R^j.a_{j\mathbf{w}}\rangle, \ldots\}$, with $i \neq j$ and $1 \leq i, j \leq k$, is the finite set of join (role) pairs (given k relations $\mathbf{R}$, $|\bowtie_{\mathbf{R}}| = k - 1$)

(2) $R^i.a_{i\mathbf{x}} \in \varrho_{R^i}$ for any $R^i \in \mathcal{R}$, and

(3) the equalities in $\bigwedge_{\bowtie_{\mathbf{R}}}$ are specified according to $\bowtie_{\mathbf{R}}$

(e.g. given $\overline{x}^1, \overline{x}^2, \overline{x}^3$ s.t. $R^1(\overline{x}^1), R^2(\overline{x}^2), R^3(\overline{x}^3)$, if $\bowtie_{\mathbf{R}} = \{(R^1.a, R^2.b), (R^2.c, R^3.d)\}$ then $\bigwedge_{\bowtie_{\mathbf{R}}} =_{\mathsf{def}} (x^1_{\tau(R^1.a)} = x^2_{\tau(R^2.b)}) \wedge (x^2_{\tau(R^2.c)} = x^3_{\tau(R^3.d)})$)

■ $\mathsf{MAND} \subseteq \wp(\varrho) \times (\mathcal{E} \cup \mathcal{V})$

□ If $\mathsf{MAND}(\{R^1.a_{11}, \ldots, R^1.a_{1n}, \ldots, R^k.a_{k1}, \ldots, R^k.a_{km}\}, O) \in \Sigma$ then

$$\forall y[A(y) \rightarrow \bigvee_{i=1}^{n} \exists \overline{z}^i (R^1(\overline{z}^i) \wedge (z^i_{\tau(R^1.a_{1i})} = y) \vee \cdots \vee \bigvee_{j=1}^{m} \exists \overline{z}^j.R^k(\overline{z}^j) \wedge (z^j_{\tau(R^k.a_{kj})} = y))]$$

■ $\mathsf{R\text{-}SET_H} \subseteq (\wp(\varrho) \times (\wp(\varrho) \times \wp(\varrho))) \times (\wp(\varrho) \times (\wp(\varrho) \times \wp(\varrho))) \times (\mu\colon \varrho \rightarrow \varrho)$ where $\mathsf{H} = \{\mathsf{Sub}, \mathsf{Exc}\}$

□ • If $\mathsf{R\text{-}SET_{Sub}}((\{R^1.a_{11}, \ldots, R^1.a_{1n}, \ldots, R^k.a_{k1}, \ldots, R^k.a_{km}\}, \bowtie_{\mathbf{R}}), (\{S^1.b_{11}, \ldots, S^1.b_{1v}, \ldots, S^q.b_{q1}, \ldots, S^q.b_{qw}\}, \bowtie_{\mathbf{S}}, \mu) \in \Sigma$ then

$$\forall \overline{y}[\exists \overline{x}^1 \ldots \overline{x}^k (\bigwedge_{j=1}^{k} R^j(\overline{x}^j) \wedge \bigwedge_{\mathbf{i1}=1}^{n} (x^1_{\tau(R^1.a_{1\mathbf{i1}})} = y_{1\mathbf{i1}}) \wedge \cdots \wedge \bigwedge_{\mathbf{ik}=1}^{m} (x^1_{\tau(R^1.a_{1\mathbf{ik}})} = y_{1\mathbf{ik}}) \wedge \bigwedge_{\bowtie_{\mathbf{R}}} (x^{r^+}_{\tau(R^{r^+}.a_{r^+\mathbf{v}_r})} = x^{r^-}_{\tau(R^{r^-}.a_{r^-\mathbf{w}_r})})) \rightarrow$$

$$\exists \overline{z}^1 \ldots \overline{z}^q (\bigwedge_{i=1}^{q} S^i(\overline{z}^i) \wedge \bigwedge_{\mathbf{i1}=1}^{n} (z^{f_{\mu(1\mathbf{i1})}}_{\tau(\mu(R^1.a_{1\mathbf{i1}}))} = y_{1\mathbf{i1}}) \wedge \cdots \wedge \bigwedge_{\mathbf{ik}=1}^{m} (z^{f_{\mu(1\mathbf{ik})}}_{\tau(\mu(R^1.a_{1\mathbf{ik}}))} = y_{1\mathbf{ik}}) \wedge \bigwedge_{\bowtie_{\mathbf{S}}} (z^{s^+}_{\tau(S^{s^+}.b_{s^+\mathbf{v}_s})} = z^{s^-}_{\tau(S^{s^-}.b_{s^-\mathbf{w}_s})}))]$$

• If $\mathsf{R\text{-}SET_{Exc}}((\{R^1.a_{11}, \ldots, R^1.a_{1n}, \ldots, R^k.a_{k1}, \ldots, R^k.a_{km}\}, \bowtie_{\mathbf{R}}), (\{S^1.b_{11}, \ldots, S^1.b_{1v}, \ldots, S^q.b_{q1}, \ldots, S^q.b_{qw}\}, \bowtie_{\mathbf{S}}, \mu) \in \Sigma$ then

$$\forall \overline{y}[\exists \overline{x}^1 \ldots \overline{x}^k (\bigwedge_{j=1}^{k} R^j(\overline{x}^j) \wedge \bigwedge_{\mathbf{i1}=1}^{n} (x^1_{\tau(R^1.a_{1\mathbf{i1}})} = y_{1\mathbf{i1}}) \wedge \cdots \wedge \bigwedge_{\mathbf{ik}=1}^{m} (x^1_{\tau(R^1.a_{1\mathbf{ik}})} = y_{1\mathbf{ik}}) \wedge \bigwedge_{\bowtie_{\mathbf{R}}} (x^{r^+}_{\tau(R^{r^+}.a_{r^+\mathbf{v}_r})} = x^{r^-}_{\tau(R^{r^-}.a_{r^-\mathbf{w}_r})})) \rightarrow$$

$$\neg(\exists \overline{z}^1 \ldots \overline{z}^q (\bigwedge_{i=1}^{q} S^i(\overline{z}^i) \wedge \bigwedge_{\mathbf{i1}=1}^{n} (z^{f_{\mu(1\mathbf{i1})}}_{\tau(\mu(R^1.a_{1\mathbf{i1}}))} = y_{1\mathbf{i1}}) \wedge \cdots \wedge \bigwedge_{\mathbf{ik}=1}^{m} (z^{f_{\mu(1\mathbf{ik})}}_{\tau(\mu(R^1.a_{1\mathbf{ik}}))} = y_{1\mathbf{ik}}) \wedge \bigwedge_{\bowtie_{\mathbf{S}}} (z^{s^+}_{\tau(S^{s^+}.b_{s^+\mathbf{v}_s})} = z^{s^-}_{\tau(S^{s^-}.b_{s^-\mathbf{w}_s})})))]$$

where:

(1) given $\varrho^{\mathsf{C_A}} = \{R^1.a_{11}, \ldots, R^1.a_{1n}, \ldots, R^k.a_{k1}, \ldots, R^k.a_{km}\}$, and $\varrho^{\mathsf{C_B}} = \{S^1.b_{11}, \ldots, S^1.b_{1v}, \ldots, S^q.b_{q1}, \ldots, S^q.b_{qw}\}$,

μ is a partial bijection s.t. for any $\langle \varrho^{\mathsf{C_A}}, \varrho^{\mathsf{C_B}}, \mu \rangle \in \mathsf{R\text{-}SET_H}$, we have $\varrho^{C_A} = \{R.a | \mu(R.a) \in \varrho^{C_B}\}$, and

(2) $f_{\mu(x\mathbf{y})} = z$ iff $\mu(R^x.a_{x\mathbf{y}}) \in \varrho_{S^z}$

First-Order Logic

■ $\mathsf{O\text{-}SET}_{\mathsf{H}} \subseteq \wp(\mathcal{E} \cup \mathcal{V}) \times \mathcal{E} \cup \mathcal{V}$ where $\mathsf{H} = \{\mathsf{Isa}, \mathsf{Tot}, \mathsf{Ex}\}$

□ • If $\mathsf{O\text{-}SET}_{\mathsf{Isa}}(\{O_1, \ldots, O_n\}, O) \in \Sigma$ then $\forall y.O_i(y) \to O(y)$ for all $i = 1, \ldots, n$

• If $\mathsf{O\text{-}SET}_{\mathsf{Tot}}(\{O_1, \ldots, O_n\}, O) \in \Sigma$ then

$$\begin{cases} \forall y.O_i(y) \to O(y) \\ \forall y.O(y) \to O_1(y) \vee \cdots \vee O_n(y), \text{ for all } i = 1, \ldots, n \end{cases}$$

• If $\mathsf{O\text{-}SET}_{\mathsf{Ex}}(\{O_1, \ldots, O_n\}, O) \in \Sigma$ then

$$\begin{cases} \forall y.O_1(y) \to O(y) \wedge \neg O_2(y) \wedge \cdots \wedge \neg O_n(y) \\ \forall y.O_2(y) \to O(y) \wedge \neg O_3(y) \wedge \cdots \wedge \neg O_{n-1}(y) \\ \quad\cdots \\ \forall y.O_{n-1}(y) \to O(y) \wedge \neg O_1(y) \\ \forall y.O_n(y) \to O(y) \end{cases}$$

■ $\mathsf{O\text{-}CARD} \subseteq (\mathcal{E} \cup \mathcal{V}) \times (\mathbb{N} \times (\mathbb{N} \cup \{\infty\}))$

□ If $\mathsf{O\text{-}CARD}(O) = (\mathsf{min}, \mathsf{max})) \in \Sigma$ then $\exists^{\geq \mathsf{min}} y.O(y) \wedge \exists^{\leq \mathsf{max}} y.O(y)$

■ $\mathsf{R\text{-}CARD} \subseteq \wp(\varrho) \times (\mathbb{N} \times (\mathbb{N} \cup \{\infty\}))$

□ If $\mathsf{R\text{-}CARD}(R.a) = (\mathsf{min}, \mathsf{max})) \in \Sigma$ then

$\exists^{\geq \mathsf{min}} x_{\tau(R.a)}.R(x_1 \ldots x_{\tau(R.a)} \ldots x_n) \wedge \exists^{\leq \mathsf{max}} x_{\tau(R.a)}.R(x_1 \ldots x_{\tau(R.a)} \ldots x_n)$

■ $\mathsf{OBJ} \subseteq \mathcal{R} \times (\mathcal{E} \cup \mathcal{V})$

□ If $\mathsf{OBJ}(R, O) \in \Sigma$ then $\forall x.O(x) \leftrightarrow \exists \overline{y}.R(\overline{y}) \wedge \mathrm{ID}^{|\varrho_R|}(\overline{y}) = x$

■ $\mathsf{RING}_{\mathsf{J}} \subseteq \wp(\varrho \times \varrho)$ where $\mathsf{J} = \{\mathsf{Irr}, \mathsf{Asym}, \mathsf{Trans}, \mathsf{Intr}, \mathsf{Antisym}, \mathsf{Acyclic}, \mathsf{Sym}, \mathsf{Ref}\}$

□ E.g. If $\mathsf{RING}_{\mathsf{Irr}}(R.a, R.b) \in \Sigma$ then $\forall x_{\tau(R.a)}, x_{\tau(R.b)}.R(x_{\tau(R.a)}, x_{\tau(R.b)}) \to \neg R(x_{\tau(R.b)}, x_{\tau(R.a)})$

■ $\mathsf{V\text{-}VAL} \colon \mathcal{V} \to \wp(\Lambda_D)$ for some $\Lambda_D \in \Lambda$ (where $\Lambda_{(\cdot)}$ associates an extension to each domain symbol)

□ If $\mathsf{V\text{-}VAL}(V) = \{d_1, \ldots, d_n\} \in \Sigma$ then $\forall x.V(x) \to (x = d_1) \vee \cdots \vee (x = d_n)$

Relational Algebra

Syntax	Semantics
$\mathsf{TYPE} \subseteq \varrho \times (\mathcal{E} \cup \mathcal{V})$	If $\mathsf{TYPE}(R.a, O) \in \Sigma$ then $\Pi_{R.a} R^{\mathcal{I}} \subseteq O^{\mathcal{I}}$
$\mathsf{FREQ} \subseteq \wp(\varrho) \times (\wp(\varrho) \times \wp(\varrho)) \times (\mathbb{N} \times (\mathbb{N} \cup \{\infty\}))$	If $\mathsf{FREQ}(\{R^1.a_{11}, \ldots, R^1.a_{1n}, \ldots, R^k.a_{k1}, \ldots, R^k.a_{km}\}, \bowtie_{\mathbf{R}}, \langle \mathsf{min}, \mathsf{max} \rangle) \in \Sigma$ then $\Pi_{\varrho^{\mathsf{C}}}(R^{1^{\mathcal{I}}} \bowtie_{\mathbf{R}} \cdots \bowtie_{\mathbf{R}} R^{k^{\mathcal{I}}}) \subseteq \{\overline{x} \mid \mathsf{min} \leq \sharp\{\sigma_{\overline{x}=\varrho^{\mathsf{C}}}(R^{1^{\mathcal{I}}} \bowtie_{\mathbf{R}} \cdots \bowtie_{\mathbf{R}} R^{k^{\mathcal{I}}})\} \leq \mathsf{max}\}$
where: (1) $\varrho^{\mathsf{C}} = \{R^1.a_{11}, \ldots, R^1.a_{1n}, \ldots, R^k.a_{k1}, \ldots, R^k.a_{km}\}$, and $\overline{x} = \varrho^{\mathsf{C}}$ iff $R^1.a_{11} = x^1_{\tau(R^1.a_{11})}, \ldots, R^k.a_{km} = x^k_{\tau(R^k.a_{km})}$ (2) $\bowtie_{\mathbf{R}} = \{\ldots, \langle R^i.a_{i\mathbf{v}} = R^j.a_{j\mathbf{w}} \rangle, \ldots\}$, with $i \neq j$ and $1 \leq i, j \leq k$, is the finite set of role pairs where the joins must be computed (e.g. given sequence of n relations, $\mathbf{R}$, $\lvert \bowtie_{\mathbf{R}} \rvert = n - 1$), and $R^x.a_{x\mathbf{y}} \in \varrho^x_{\mathcal{R}}$ for any $R^x \in \mathcal{R}$	
$\mathsf{MAND} \subseteq \wp(\varrho) \times (\mathcal{E} \cup \mathcal{V})$	If $\mathsf{MAND}(\{R^1.a_{11}, \ldots, R^1.a_{1n}, \ldots, R^k.a_{k1}, \ldots, R^k.a_{km}\}, O) \in \Sigma$ then $O^{\mathcal{I}} \subseteq \Pi_{R^1.a_{11}} R^{1^{\mathcal{I}}} \cup \cdots \cup \Pi_{R^1.a_{1n}} R^{1^{\mathcal{I}}} \cup \cdots \cup \Pi_{R^k.a_{k1}} R^{k^{\mathcal{I}}} \cup \cdots \cup \Pi_{R^k.a_{km}} R^{k^{\mathcal{I}}}$
$\mathsf{R\text{-}SET_H} \subseteq ((\wp(\varrho) \times (\wp(\varrho) \times \wp(\varrho))) \times (\wp(\varrho) \times (\wp(\varrho) \times \wp(\varrho))) \times (\mu \colon \varrho \to \varrho)$	• If $\mathsf{R\text{-}SET_{Sub}}((\{R^1.a_{11}, \ldots, R^1.a_{1n}, \ldots, R^k.a_{k1}, \ldots, R^k.a_{km}\}, \bowtie_{\mathbf{R}}),$ $(\{S^1.b_{11}, \ldots, S^1.b_{1v}, \ldots, S^q.b_{q1}, \ldots, S^q.b_{qw}\}, \bowtie_{\mathbf{S}}, \mu) \in \Sigma$ then $\Pi_{\varrho^{\mathsf{C_A}}}(R^{1^{\mathcal{I}}} \bowtie_{\mathbf{R}} \cdots \bowtie_{\mathbf{R}} R^{k^{\mathcal{I}}}) \subseteq \Pi_{\varrho^{\mathsf{C_B}}}(S^{1^{\mathcal{I}}} \bowtie_{\mathbf{S}} \cdots \bowtie_{\mathbf{S}} S^{q^{\mathcal{I}}})$ • If $\mathsf{R\text{-}SET_{Exc}}((\{R^1.a_{11}, \ldots, R^1.a_{1n}, \ldots, R^k.a_{k1}, \ldots, R^k.a_{km}\}, \bowtie_{\mathbf{R}}),$ $(\{S^1.b_{11}, \ldots, S^1.b_{1v}, \ldots, S^q.b_{q1}, \ldots, S^q.b_{qw}\}, \bowtie_{\mathbf{S}}, \mu) \in \Sigma$ then $\Pi_{\varrho^{\mathsf{C_A}}}(R^{1^{\mathcal{I}}} \bowtie_{\mathbf{R}} \cdots \bowtie_{\mathbf{R}} R^{k^{\mathcal{I}}}) \cap \Pi_{\varrho^{\mathsf{C_B}}}(S^{1^{\mathcal{I}}} \bowtie_{\mathbf{S}} \cdots \bowtie_{\mathbf{S}} S^{q^{\mathcal{I}}}) = \varnothing$
where: (1) $\varrho^{\mathsf{C_A}} = \{R^1.a_{11}, \ldots, R^1.a_{1n}, \ldots, R^k.a_{k1}, \ldots, R^k.a_{km}\}$, and $\varrho^{\mathsf{C_B}} = \{S^1.b_{11}, \ldots, S^1.b_{1v}, \ldots, S^q.b_{q1}, \ldots, S^q.b_{qw}\}$ (2) μ is a partial bijection s.t. for any $\langle \varrho^{\mathsf{C_A}}, \varrho^{\mathsf{C_B}}, \mu \rangle \in \mathsf{R\text{-}SET_H}$, we have $\varrho^{C_A} = \{R.a \mid \mu(R.a) \in \varrho^{C_B}\}$, and (3) $\mathsf{H} = \{\mathsf{Sub}, \mathsf{Exc}\}$	
$\mathsf{O\text{-}SET_H} \subseteq \wp(\mathcal{E} \cup \mathcal{V}) \times \mathcal{E} \cup \mathcal{V}$ where $\mathsf{H} = \{\mathsf{Isa}, \mathsf{Tot}, \mathsf{Ex}\}$	• If $\mathsf{O\text{-}SET_{Isa}}(\{O_1, \ldots, O_n\}, O) \in \Sigma$ then $O_i^{\mathcal{I}} \subseteq O^{\mathcal{I}}$ for $1 \leq i \leq n$ • If $\mathsf{O\text{-}SET_{Tot}}(\{O_1, \ldots, O_n\}, O) \in \Sigma$ then $O^{\mathcal{I}} \subseteq \bigcup_{i=1}^{n} O_i^{\mathcal{I}}$ • If $\mathsf{O\text{-}SET_{Ex}}(\{O_1, \ldots, O_n\}, O) \in \Sigma$ then $\mathsf{O\text{-}SET_{Isa}}(\{O_1, \ldots, O_n\}, O) \in \Sigma$ and $O_i^{\mathcal{I}} \cap O_j^{\mathcal{I}} = \varnothing$ for any $1 \leq i < j \leq n$
$\mathsf{O\text{-}CARD} \subseteq (\mathcal{E} \cup \mathcal{V}) \times (\mathbb{N} \times (\mathbb{N} \cup \{\infty\}))$	If $\mathsf{O\text{-}CARD}(O) = (\mathsf{min}, \mathsf{max}) \in \Sigma$ then $\mathsf{min} \leq \sharp\{o \mid o \in O^{\mathcal{I}}\} \leq \mathsf{max}$
$\mathsf{R\text{-}CARD} \subseteq \wp(\varrho) \times (\mathbb{N} \times (\mathbb{N} \cup \{\infty\}))$	If $\mathsf{R\text{-}CARD}(R.a) = (\mathsf{min}, \mathsf{max}) \in \Sigma$ then $\mathsf{min} \leq \sharp\{o \mid o \in \Pi_{R.a} R^{\mathcal{I}}\} \leq \mathsf{max}$
$\mathsf{OBJ} \subseteq \mathcal{R} \times (\mathcal{E} \cup \mathcal{V})$	If $\mathsf{OBJ}(R, O) \in \Sigma$ then $\mathrm{ID}^{\mathcal{I}}(R^{\mathcal{I}}) = O^{\mathcal{I}}$
$\mathsf{RING_J} \subseteq \wp(\varrho \times \varrho)$ where $\mathsf{J} = \{\mathsf{Irr}, \mathsf{Asym}, \mathsf{Trans}, \mathsf{Intr}, \mathsf{Antisym}, \mathsf{Acyclic}, \mathsf{Sym}, \mathsf{Ref}\}$	If $\mathsf{RING_J}(R.a, R.b) \in \Sigma$ then $\Pi_{(R.a,R.b)} R^{\mathcal{I}}$ is *irreflexive, asymmetric, transitive, intransitive,* *antisymmetric, acyclic, symmetric, reflexive*
$\mathsf{V\text{-}VAL} \colon \mathcal{V} \to \wp(\Lambda_D)$ for some $\Lambda_D \in \Lambda$	If $\mathsf{V\text{-}VAL}(V) = \{v_1^D, \ldots, v_n^D\} \in \Sigma$ then $V^{\mathcal{I}} = \{v_1^D, \ldots, v_n^D\}$ for some D

A correct encoding in OWL2

Background domain axioms:	$E_i \sqsubseteq \neg(D_1 \sqcup \cdots \sqcup D_l)$ for $i \in \{1,\ldots,n\}$ $V_i \sqsubseteq D_j$ for $i \in \{1,\ldots,m\}$, and some j with $1 \leq j \leq l$ $D_i \sqsubseteq \sqcap_{j=i+1}^{l} \neg D_j$ for $i \in \{1,\ldots,l\}$ $\top \sqsubseteq A_{\top_1} \sqcup \cdots \sqcup A_{\top_{n_{max}}}$ $\top \sqsubseteq (\leq 1i.\top)$ for $i \in \{1,\ldots,n_{max}\}$ $\forall i.\bot \sqsubseteq \forall i+1.\bot$ for $i \in \{1,\ldots,n_{max}\}$ $A_{\top_n} \equiv \exists 1.A_{\top_1} \sqcap \cdots \sqcap \exists n.A_{\top_1} \sqcap \forall n+1.\bot$ for $n \in \{2,\ldots,n_{max}\}$ $A_R \sqsubseteq A_{\top_n}$ for each atomic relation R of arity n $A \sqsubseteq A_{\top_1}$ for each atomic concept A
$\mathsf{TYPE}(R.a, O)$	$\exists\tau(R.a)^-.A_R \sqsubseteq O$
$\mathsf{FREQ}^-(R.a, \langle\mathsf{min},\mathsf{max}\rangle)$	$\exists\tau(R.a)^-.A_R \sqsubseteq\ \geq \mathsf{min}\ \tau(R.a)^-.A_R \sqcap \leq \mathsf{max}\ \tau(R.a)^-.A_R$
$\mathsf{MAND}(\{R^1.a_1,\ldots,R^1.a_n,$ $\ldots,R^k.a_1,\ldots,R^k.a_m\}, O)$	$O \sqsubseteq \exists\tau(R^1.a_1)^-.A_{R^1} \sqcup \cdots \sqcup \exists\tau(R^1.a_n)^-.A_{R^1} \sqcup \cdots \sqcup$ $\exists\tau(R^k.a_1)^-.A_{R^k} \sqcup \cdots \sqcup \exists\tau(R^k.a_m)^-.A_{R^k}$
$^{(A)}$ $\mathsf{R\text{-}SET}^-_{\mathsf{Sub}}(A,B)$ $^{(A)}$ $\mathsf{R\text{-}SET}^-_{\mathsf{Exc}}(A,B)$	$A_R \sqsubseteq A_S$ $^{(A)}\ A = \{R.a_1,\ldots,R.a_n\}, B = \{S.b_1,\ldots,S.b_n\}$ $A_R \sqsubseteq A_{\top_n} \sqcap \neg A_S$
$^{(B)}$ $\mathsf{R\text{-}SET}^-_{\mathsf{Sub}}(A,B)$ $^{(B)}$ $\mathsf{R\text{-}SET}^-_{\mathsf{Exc}}(A,B)$	$\exists\tau(R.a_i)^-.A_R \sqsubseteq \exists\tau(S.b_j)^-.A_S$ $^{(B)}\ A = \{R.a_i\}, B = \{S.b_j\}$ $\exists\tau(R.a_i)^-.A_R \sqsubseteq A_{\top_n} \sqcap \neg\exists\tau(S.b_j).A_S$
$\mathsf{O\text{-}SET}_{\mathsf{Isa}}(\{O_1,\ldots,O_n\}, O)$	$O_1 \sqcup \cdots \sqcup O_n \sqsubseteq O$
$\mathsf{O\text{-}SET}_{\mathsf{Tot}}(\{O_1,\ldots,O_n\}, O)$	$O \sqsubseteq O_1 \sqcup \cdots \sqcup O_n$
$\mathsf{O\text{-}SET}_{\mathsf{Ex}}(\{O_1,\ldots,O_n\}, O)$	$O_1 \sqcup \cdots \sqcup O_n \sqsubseteq O$ and $O_i \sqsubseteq \sqcap_{j=i+1}^{n} \neg O_j$ for each $i = 1,\ldots,n$
$\mathsf{OBJ}(R, O)$	$O \equiv A_R$

Future work

- Extend the expressivity of the captured ORM2 fragment
- A tighter integration of our plugin with NORMA (cooperation???)
- A higher-level plugin implementing the well-founded methodology for formal ontology design based on the work of Guarino et al
- (a complete version of our work can be found in a technical report)